basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P3/*WISKUNDE V3*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2014

MEMORANDUM

MARKS/*PUNTE*: 100

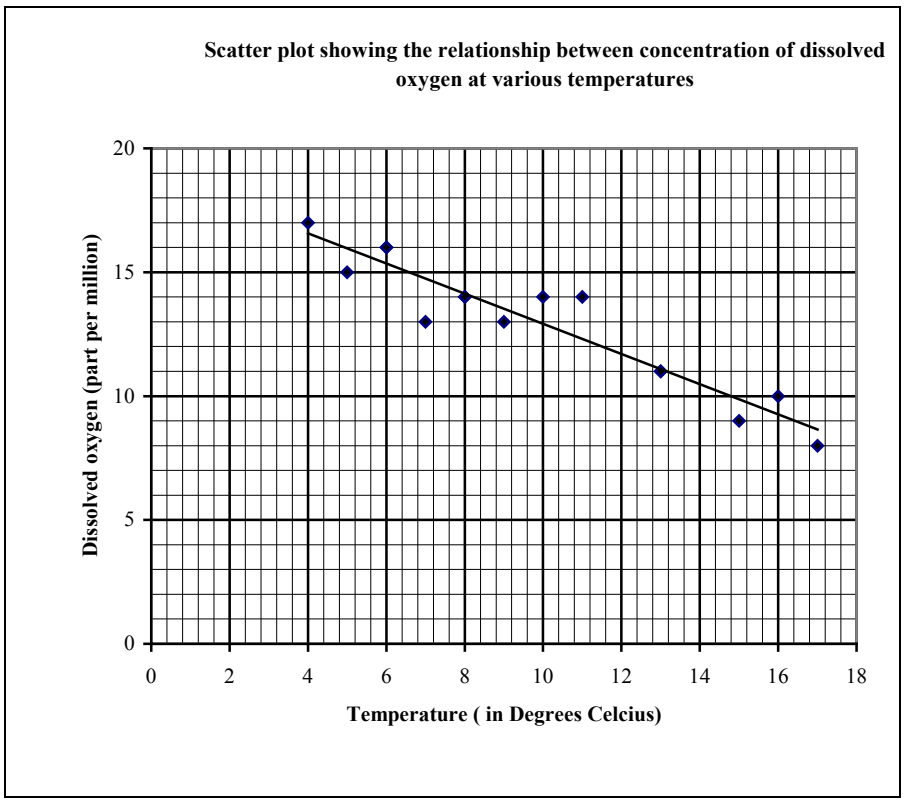
This memorandum consists of 11 pages.
Hierdie memorandum bestaan uit 11 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the first attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE keer beantwoord, merk net die eerste poging.*
- *As 'n kandidaat 'n antwoord deurhaal en nie oordoen nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid moet DEURGAANS in die memorandum toegepas word.*

QUESTION/*VRAAG* 1

Temperature (°C) (x)	17	15	13	16	11	13	10	8	6	7	8	4	5	9	6
DO (ppm) (y)	8	9	11	10	14	11	14	14	16	13	14	17	15	13	16

1.1 & 1.3		1.1 ✓ 1 – 4 points correct ✓ 5 – 9 points correct ✓ all 12 points correct (3) 1.3 ✓ correct gradient ✓ passing close to (8 ; 14) and (13 ; 11) (2)
1.2	$a = 19{,}01$ (19,00889878…) $b = -0{,}61$ ($-0{,}6090100111$…) $y = 19{,}01 - 0{,}61x$	✓✓$a = 19{,}01$ ✓ $b = -0{,}61$ ✓ $y = 19{,}01 - 0{,}61x$ (4)

1.4	$\hat{y} = 19{,}01 - 0{,}61(14)$ $= 10{,}47$ **OR** If calculator is used $\hat{y} = 10{,}48$ (10,48275862…) **OR** If least squares method is used or the graph is used, $\hat{y} = 10{,}5$	✓ substitute $x = 14$ ✓ answer (2) ✓✓answer (2) ✓✓answer (2)
1.5	$r = -0{,}94$ (– 0 ,9429488543…)	✓✓answer (2)
1.6	There exists a very strong negative correlation between the variables. As the temperature in the lake water increases, so the concentration of dissolved oxygen decreases. *Daar bestaan 'n sterk negatiewe korrelasie tussen die veranderlikes. Soos wat die temperatuur van die water in die meer verhoog, so verlaag die konsentrasie van die opgeloste suurstof.*	✓strong ✓ negative (2) **[15]**

QUESTION/*VRAAG* 2

2.1	$a = 73$ $b = 42$ $c = 107$ $d = 68$	✓ $a = 73$ ✓ $b = 42$ ✓ $c = 107$ ✓ $d = 68$ (4)

	Aged < 40	**Aged ≥ 40**	**Totals**
Liked the movie	65	37	102
Did not like the movie	***b = 42***	31	***a = 73***
Totals	***c = 107***	***d = 68***	175

2.2	P(less than 40 and did not like the movie) = $\frac{42}{175}$ (0,24)	✓ 42 ✓175 (2)
2.3	P(less than 40 and liked the movie) $= \frac{65}{175} = 0{,}37$ (0,3714285714…) P(Age less than 40) = $\frac{107}{175}$ P(Critic liked the movie) = $\frac{102}{175}$ P(Age less than 40) × P(Critic liked the movie) $= \frac{107}{175} \times \frac{102}{175} = 0{,}36$ (0,3563755102…) Since P(less than 40 and liked the movie) ≠ P(Age less than 40) × P(Critic liked the movie), we can conclude that the events are not independent/*nie onafhanklik nie*.	✓ P(less than 40 and liked the movie) = $\frac{65}{175}$ ✓ P(Age less than 40) = $\frac{107}{175}$ **OR** P(Critic liked the movie) = $\frac{102}{175}$ ✓ P(Age less than 40) × P(Critic liked the movie)= 0,36 ✓conclusion (4)

	OR P(less than 40 and did not like the movie) = $\frac{42}{175} = 0,24$ P(Age less than 40) = $\frac{107}{175}$ P(Critic did not like the movie) = $\frac{73}{175}$ P(Age less than 40) × P(Critic did not like the movie) = $\frac{107}{175} \times \frac{73}{175} = 0,255$ (0,2550530612…) Since P(less than 40 and did not like the movie) ≠ P(Age less than 40) × P(Critic did not like the movie), we can conclude that the events are not independent/*nie onafhanklik nie*.	✓ P(less than 40 and did not like the movie) = $\frac{42}{175}$ ✓ P(Age less than 40) = $\frac{107}{175}$ **OR** P(Critic did not like the movie) = $\frac{73}{175}$ ✓ P(Age less than 40) × P(Critic liked the movie)= 0,255 ✓conclusion (4) **[10]**

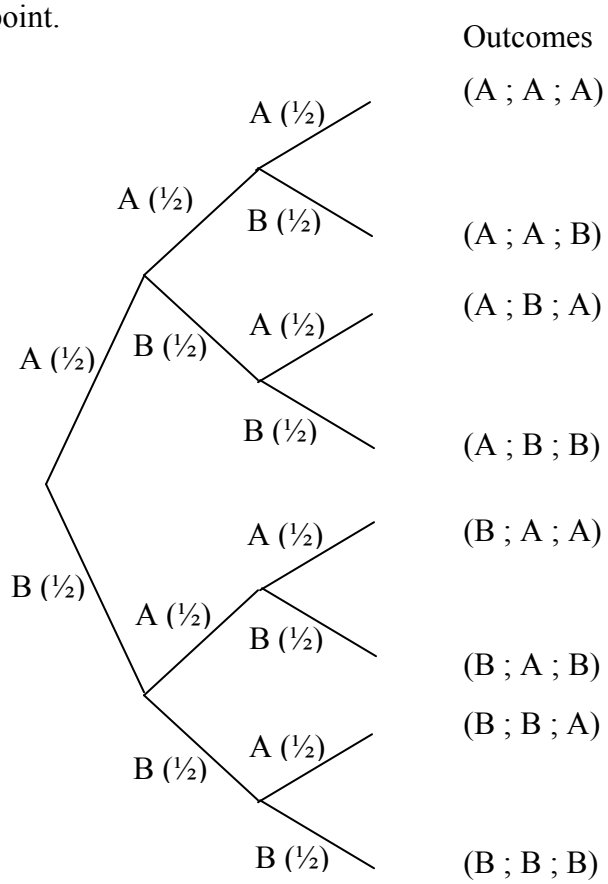
QUESTION/*VRAAG* 3

3.1	The interval 500 to 542 hours represents 48% of the data. $\Rightarrow$ 542 is at 2 standard deviations to the right of the mean. $542 = 500 + 2\sigma$ $2\sigma = 42$ $\sigma = 21$	✓ 2 standard deviations ✓21 (2)
3.2	458 is at 2 standard deviations to the left of the mean. area between mean and 2 standard deviations = 48% 521 is at 1 standard deviation to the right of the mean. area between mean and 1 standard deviation = 34% $\therefore$ Total area between 458 and 521 hours = 48% + 34% = 82%	✓ 48% ✓ 34% ✓82% (3)
3.3	The expected minimum lifetime will occur at 3 standard deviations to the left of the mean. $\therefore$ Expected minimum lifetime $= 500 - 3(21)\sigma = 437$ hours	✓ 3 standard deviations ✓437 (2) **[7]**

QUESTION/*VRAAG* 4

4.1	Different ways for 8 learners to be seated $= 8!$ $= 40\ 320$	✓ 8 ✓40 320 (2)
4.2	Consider the 3 learners as a single entity. These 3 learners can be seated in $3! = 6$ different ways. Now this group of 3 and the remaining 5 learners can be seated in $6! = 720$ different ways. In total there are $6 \times 720 = 4320$ different ways for the 3 learners to be seated together. $3!6!$ $=4\ 320$	✓ 6 ✓ 720 ✓4 320 (3)
4.3	First let us consider the different number of ways that these 2 learners can be seated next to one another. This can be done in $2! \times 7! = 10\ 080$ different ways. Now these two learners may not be seated next to one another in $40\ 320 - 10\ 080$ $= 30\ 240$ different ways. **OR/OF** Let person A sit at the end of the row and person B not sit next to person A. This can be done in $1 \times 6 \times 6! \times 2$ different ways A | | | | | | | | ↑ not B Let person A not sit at the end of the row. this can be done in $1 \times 6 \times 5 \times 5! \times 6$ different ways | A | | | | | | | ↑ ↑ not B not B In total we have $1 \times 6 \times 6! \times 2 + 1 \times 6 \times 5 \times 5! \times 6$ $= 30\ 240$ different ways	✓ $2! \times 7!$ or 10 080 ✓ $40\ 320 - 10\ 080$ ✓ answer (3) ✓ $1 \times 6 \times 6! \times 2 = 8\ 640$ ✓ $1 \times 6 \times 5 \times 5! \times 6 = 21\ 600$ ✓ answer (3) **[8]**

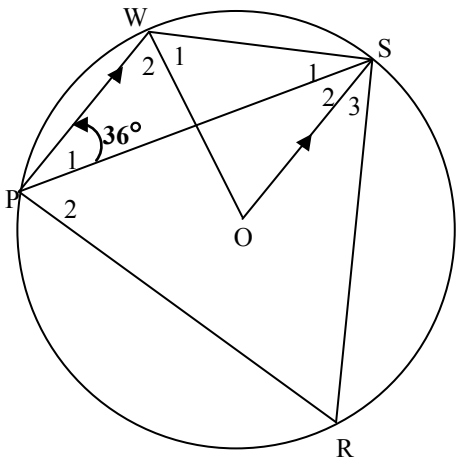
QUESTION/*VRAAG* 5

5.1	Let A represent Alfred winning a point and B represent Barry winning a point.  	✓ first branch correct ✓ second branch correct ✓ third branch correct ✓ probabilities at each branch ✓ all outcomes listed (5)
5.2	P(Barry wins three points) = $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$	✓ $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ ✓ $\frac{1}{8}$ (2)
5.3	P(Alfred wins two points and Barry wins one point) = P(A; A; B) + P(A; B; A) + P(B; A; A) $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $=\frac{3}{8}$	✓ addition of $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ ✓ $\frac{3}{8}$ (2)
5.4	P(Alfred wins 3 of the four points) = P(AAAB) + P(AABA) + P(ABAA) + P(BAAA) $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $=4\left(\frac{1}{2}\right)^4$ $=\frac{1}{4}$	✓✓ P(AAAB) + P(AABA) + P(ABAA) + P(BAAA) ✓ $4\left(\frac{1}{2}\right)^4$ ✓ answer (4) **[13]**

QUESTION/*VRAAG* 6

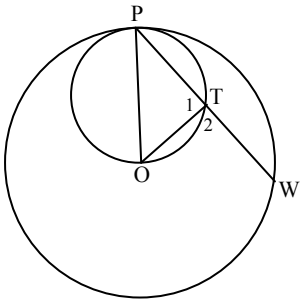
6.1	By inspection $T_{n+1} = T_n + T_{n-1} - 2;\ T_1 = 4,\ T_2 = 7;\ n \geq 1,\ n \in N$ $a = 1$ $b = -2$ **OR/*OF*** $T_{n+1} = T_n + aT_{n-1} + b$ $9 = 7 + 4a + b$ $2 = 4a + b \quad \ldots (1)$ $14 = 9 + 7a + b$ $5 = 7a + b \quad \ldots (2)$ $(2) - (1): \quad 3a = 3$ $a = 1$ $b = -2$	✓✓✓✓ Answer only: full marks (4) ✓ $2 = 4a + b$ ✓ $5 = 7a + b$ ✓ $a = 1$ ✓ $b = -2$ (4)
6.2	$T_7 = 52$	✓ answer (1) **[5]**

QUESTION/*VRAAG* 7



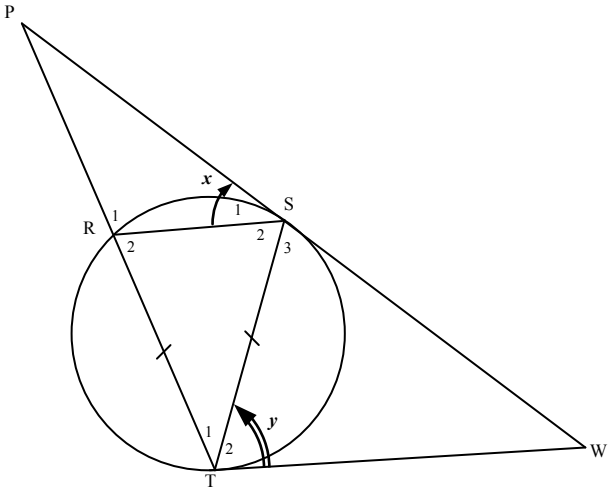
7.1	$S\hat{O}W = 72°$ (∠ circ cent = 2 ∠ circumference) (*middelpunts∠ = 2 omtreks ∠)*	✓ $S\hat{O}W = 72°$ ✓ ∠ circ cent = 2 ∠ circumference (2)
7.2	$\hat{W}_2 = 72°$ (alt ∠s; PW || SA) / *(verw ∠e; PW || SA)*	✓ $\hat{W}_2 = 72°$ ✓ PW || SO (2)
7.3	$O\hat{S}W = \hat{W}_1$ (∠s opp = radii)/*(∠e teenoor = radiusse)* $2O\hat{S}W + 72° = 180°$ (∠ sum Δ)/*(som van binne∠e Δ)* $2O\hat{S}W = 108°$ $O\hat{S}W = 54°$	✓ $O\hat{S}W = \hat{W}_1$ ✓ ∠s opp = radii ✓ answer (3)
7.4	$\hat{R} + \hat{W}_1 + \hat{W}_2 = 180°$ (opp ∠s cyclic quad)/*(oorst ∠e koordevierhoek)* $\hat{R} + 54° + 72° = 180°$ $\hat{R} = 54°$	✓ $\hat{R} + \hat{W}_1 + \hat{W}_2 = 180°$ ✓ opp ∠s cyclic quad ✓ answer (3) **[10]**

QUESTION/*VRAAG* 8



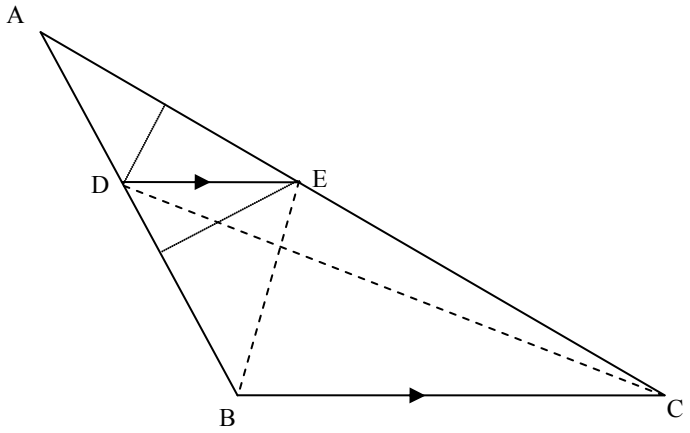
8.	$\hat{T}_1 = 90°$ (∠s in a semi-circle)/*(∠e in 'n halwe sirkel)* PT = TW = 24 cm (line from circ cent ⊥ ch) *(lyn van middelpunt ⊥ koord)* $OP^2 = OT^2 + PT^2$ (Pythagoras) $OP^2 = (10)^2 + (24)^2$ $OP^2 = 676$ OP = 26 cm Radius of smaller circle = 13 cm	✓ $\hat{T}_1 = 90°$ ✓ ∠s in a semi-circle ✓ line from circ cent ⊥ ch ✓ OP = 26 cm ✓ answer **[5]**

QUESTION/*VRAAG* 9

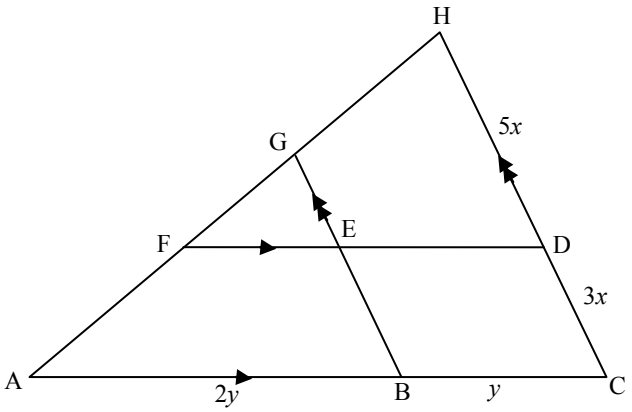


9.1	$\hat{R}_2 = y$ (tan ch th) *(hoek tussen raaklyn en koord)* RT = ST (given) $\hat{S}_2 = y$ (∠s opp = sides)/*(∠e teenoor = sye)* SW = WT (tan from common point)/*(raaklyn vanaf selfde punt)* $\hat{S}_3 = y$ (∠s opp = sides)/*(∠e teenoor = sye)* **OR** SW = WT (tan from common point)/*(raaklyn vanaf selfde punt)* $\hat{S}_3 = y$ (∠s opp = sides)/*(∠e teenoor = sye)* $\hat{R}_2 = \hat{S}_3 = y$ (tan ch th)/*(hoek tussen raaklyn en koord)* RT = ST (given) $\hat{S}_2 = y$ (∠s opp = sides)/*(∠e teenoor = sye)*	✓ $\hat{R}_2 = y$ ✓ tan ch th ✓ $\hat{S}_2 = y$ ✓ ∠s opp = sides ✓ tan from common point ✓ $\hat{S}_3 = y$ (6) ✓ tan from common point ✓ $\hat{S}_3 = y$ ✓ $\hat{R}_2 = y$ ✓ tan ch th ✓ $\hat{S}_2 = y$ ✓ ∠s opp = sides (6)
9.2	In ΔPRS and ΔPST i. $\hat{P}$ is common ii. $\hat{T}_1 = \hat{S}_1 = x$ (tan ch th)/*(hoek tussen raaklyn en koord)* iii. $\hat{R}_1 = P\hat{S}T = x + y$ (3^{rd} ∠ of the Δ) ΔPRS ||| ΔPST (∠∠∠)	✓ $\hat{P}$ is common ✓ $\hat{T}_1 = \hat{S}_1 = x$ ✓ $\hat{R}_1 = P\hat{S}T = x + y$ **OR** (∠∠∠) (3)
9.3	$\frac{PS}{PT} = \frac{RS}{ST}$ (||| Δs) ST = RT (given) $\frac{PS}{PT} = \frac{RS}{RT}$ PS × RT = RS × PT	✓ $\frac{PS}{PT} = \frac{RS}{ST}$ ✓ ||| Δs ✓ ST = RT (3) **[12]**

QUESTION 10



10.1	Join E to B and D to C.	✓ construction
	$\frac{\text{area } \Delta AED}{\text{area } \Delta DEB} = \frac{AD}{DB}$ (common altitudes)/ (*gemeenskaplike hoogtelyne)*	✓ $\frac{\text{area } \Delta AED}{\text{area } \Delta DEB} = \frac{AD}{DB}$ ✓ = alts ∴ ratio areas = ratio bases
	$\frac{\text{area } \Delta AED}{\text{area } \Delta DEC} = \frac{AD}{EC}$ (common altitudes)	✓ $\frac{\text{area } \Delta AED}{\text{area } \Delta DEC} = \frac{AD}{EC}$
	area ΔAED is common	
	area ΔDEB = area ΔDEC (DE || BC; same base BC)	✓ area ΔDEB = area ΔDEC and ✓ DE || BC; same base BC
	$\frac{\text{area } \Delta AED}{\text{area } \Delta DEB} = \frac{\text{area } \Delta AED}{\text{area } \Delta DEC}$ $\frac{AD}{DB} = \frac{AD}{EC}$	✓ $\frac{\text{area } \Delta AED}{\text{area } \Delta DEB} = \frac{\text{area } \Delta AED}{\text{area } \Delta DEC}$ (7)



10.2.1	$\frac{HF}{AH}=\frac{5}{8}$ (FD || AC; Prop Th/*Verhouding St*) $HF=\frac{5}{8}AH$ $HF=\frac{5}{8}(48)$ $HF = 30$ cm	✓ $\frac{HF}{AH}=\frac{5}{8}$ ✓ FD || AC ✓ answer (3)
10.2.2	AF = 18 cm $\frac{AF+FG}{HF-FG}=\frac{2}{1}$ (BG || CH; Prop Th/*Verhouding St*) $\frac{18+FG}{30-FG}=2$ $18+FG=2(30-FG)$ $18+FG=60-2FG$ $3FG=42$ $FG=14$ cm **OR** $\frac{GH}{AH}=\frac{BC}{AC}$ (BG || CH; Prop Th/*Verhouding St*) $\frac{GH}{48}=\frac{1}{3}$ GH = 16 cm FG = HF – FG = 30 – 16 = 14 cm	✓ AF = 18 ✓ $\frac{AF+FG}{HF-FG}=\frac{2}{1}$ ✓ answer (3) ✓ $\frac{GH}{AH}=\frac{BC}{AC}$ ✓ GH = 16 cm ✓ answer (3)
10.2.3	EF : ED = GF : GH (BG || CH; Prop Th/*Verhouding St*) EF : ED = 14 : 16 = 7 : 8	✓ EF : ED = GF : GH ✓ answer (2) **[15]**

TOTAL/*TOTAAL*: 100